Section : PART-A General Aptitude

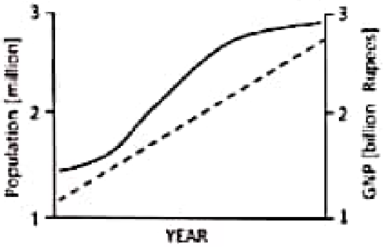
Q.1 Water drips out of the bottom of a cylindrical bucket that is initially full. The rate of dripping is proportional to the height of water column in the bucket. If the rate of dripping at half height is R, then the average rate of dripping until the bucket becomes almost empty, is

1. greater than R
2. R
3. between $R/2$ and R
4. less than $R/2$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932065
Option 1 ID : 3398938029
Option 2 ID : 3398938030
Option 3 ID : 3398938031
Option 4 ID : 3398938032
Status : Not Answered
Chosen Option : --

Q.2 The graph shows the population (*solid line*) and the Gross National Product (GNP) (*dash line*) of a country over several years.



Which of the following statements can be inferred from the graph?

1. The rate of change is the same for both, the population and the GNP
2. The population has grown faster than the GNP over the entire period
3. Per capita income at the end of the period is greater than at the beginning
4. Per capita income increased because of growth in the population

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932058
Option 1 ID : 3398938001
Option 2 ID : 3398938002
Option 3 ID : 3398938003
Option 4 ID : 3398938004
Status : Not Answered
Chosen Option : --

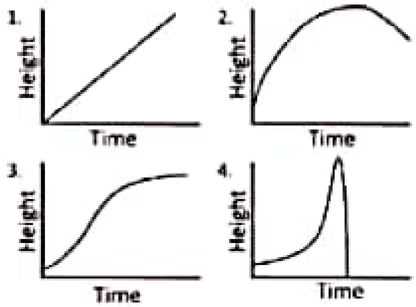
Q.3 In an examination, if more than 15 questions are attempted, only the first 15 attempted ones are evaluated. Answers are awarded + 2 marks if correct and – 0.5 if wrong. A candidate answers 19 questions and gets 15 marks. How many questions are answered correctly in the first fifteen attempted?

1. 9
2. 10
3. 11
4. 12

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932063
Option 1 ID : 3398938021
Option 2 ID : 3398938022
Option 3 ID : 3398938023
Option 4 ID : 3398938024
Status : Not Answered
Chosen Option : --

Q.4 Which of the following graphs correctly shows the growth of a sapling to a mature tree?



Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932066
Option 1 ID : 3398938033
Option 2 ID : 3398938034
Option 3 ID : 3398938035
Option 4 ID : 3398938036
Status : Not Answered
Chosen Option : --

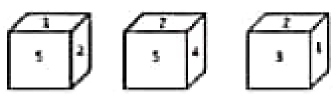
Q.5 "The father of my biological son is the only child of your parents."
The statement can be true

1. under no condition
2. only if a woman says so
3. only if a married man says so
4. only if an unmarried man says so

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932075
Option 1 ID : 3398938069
Option 2 ID : 3398938070
Option 3 ID : 3398938071
Option 4 ID : 3398938072
Status : Not Answered
Chosen Option : --

Q.6 The following are the three positions of a die with numbers 1 to 6 written on its faces. Then the number 6 appears on the face opposite that of



1. 1
2. 2
3. 4
4. 5

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932070
Option 1 ID : 3398938049
Option 2 ID : 3398938050
Option 3 ID : 3398938051
Option 4 ID : 3398938052
Status : Answered
Chosen Option : 2

Q.7 An LED bulb costs 5 times as much as a filament bulb but consumes only 1/5 of electrical power. If C is the cost of the filament bulb and P is the electric power tariff, the extra cost of the LED bulb will be recovered (in comparison to the filament bulb) in usage time of

1. 5 years
2. 5 C/P
3. C/5P
4. C/P

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932067
Option 1 ID : 3398938037
Option 2 ID : 3398938038
Option 3 ID : 3398938039
Option 4 ID : 3398938040
Status : Not Answered
Chosen Option : --

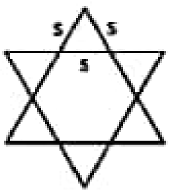
Q.8 The straight line $y_1 = 4$ and the curve $y_2 = \sin(4x)$, with x varying from 0 to 450.

1. do not intersect at all.
2. touch each other at only one point.
3. intersect at 5 points.
4. intersect at 10 points.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932064
Option 1 ID : 3398938025
Option 2 ID : 3398938026
Option 3 ID : 3398938027
Option 4 ID : 3398938028
Status : Not Answered
Chosen Option : --

Q.9 What is the area of the figure shown below, assuming all small triangles to be equilateral triangles of side S?



1. $3\sqrt{3}\,S^2$
2. $6\,S^2$
3. $4\sqrt{3}\,S^2$
4. $3\,S^2$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932068
Option 1 ID : 3398938041
Option 2 ID : 3398938042
Option 3 ID : 3398938043
Option 4 ID : 3398938044
Status : Not Answered
Chosen Option : --

Q.10 Person X purchases a house at a price P and sells it to Y at a profit of 10%. Y sells it back to X incurring a loss of 10%. As a result

1. X makes a profit of 11 % of P
2. Y incurs a loss of 10% of P
3. Y makes a profit of 11 % of P
4. Neither X nor Y gains or loses

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **3398932060**
Option 1 ID : **3398938009**
Option 2 ID : **3398938010**
Option 3 ID : **3398938011**
Option 4 ID : **3398938012**
Status : **Not Answered**
Chosen Option : --

Q.11 A unicellular organism reproduces by cell division. When two of the organisms come together they tend to destroy each other. If n is the number of cells, which of the following equations best represents the rate of change of the population [where α and β are positive constants]?

1. $\frac{dn}{dt} = \alpha n - \beta n^2$
2. $\frac{dn}{dt} = -\alpha n - \beta n^2$
3. $\frac{dn}{dt} = \alpha n + \beta n^2$
4. $\frac{dn}{dt} = -\alpha n + \beta n^2$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **3398932061**
Option 1 ID : **3398938013**
Option 2 ID : **3398938014**
Option 3 ID : **3398938015**
Option 4 ID : **3398938016**
Status : **Not Answered**
Chosen Option : --

Q.12 Three metal cubes whose diagonals are $\sqrt{3}, 6\sqrt{3}$ and $8\sqrt{3}$ units respectively, are melted to make a new cube. The side of the new cube would be

1. 15 units
2. $10\sqrt{3}$ units
3. $15\sqrt{3}$ units
4. 9 units

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **3398932069**
Option 1 ID : **3398938045**
Option 2 ID : **3398938046**
Option 3 ID : **3398938047**
Option 4 ID : **3398938048**
Status : **Not Answered**
Chosen Option : --

Q.13 Governor : State : :

1. ship : captain
2. admiral : navy
3. jail : prisoner
4. student : teacher

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **3398932073**
Option 1 ID : **3398938061**
Option 2 ID : **3398938062**
Option 3 ID : **3398938063**
Option 4 ID : **3398938064**
Status : **Answered**
Chosen Option : **2**

Q.14 Birds beat their wings up and down but propel themselves forward because

1. the neck is held at an angle with the body axis
2. the flight feathers make an angle with the wing axis
3. their body is streamlined
4. their tail is fan shaped

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **3398932074**
Option 1 ID : **3398938065**
Option 2 ID : **3398938066**
Option 3 ID : **3398938067**
Option 4 ID : **3398938068**
Status : **Not Answered**
Chosen Option : **--**

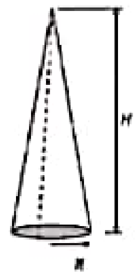
Q.15 In a group of researchers Biologists are thrice as many as Chemists whereas the Physicists are twice as many as Biologists. If there are 5 Chemists in the group, what is the total number of these three types of researchers?

1. 30
2. 45
3. 50
4. 75

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **3398932059**
Option 1 ID : **3398938005**
Option 2 ID : **3398938006**
Option 3 ID : **3398938007**
Option 4 ID : **3398938008**
Status : **Answered**
Chosen Option : **3**

Q.16 A hollow paper cone of height H and radius R is cut along the dotted line and opened to form a sector of a circle. The angle subtended by the sector (in radians) is



1. $2\pi\frac{R}{H}$
2. $2\pi\frac{H}{\sqrt{R^2+H^2}}$
3. $2\pi\frac{R}{\sqrt{R^2+H^2}}$
4. $2\pi\frac{H}{R}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932077
Option 1 ID : 3398938077
Option 2 ID : 3398938078
Option 3 ID : 3398938079
Option 4 ID : 3398938080
Status : Not Answered
Chosen Option : --

Q.17 Among the following pairs of numbers, the smallest difference is between

1. 1.0×10^{23} and 1.0×10^{-23}
2. 1.0×10^{-23} and -1.0×10^{-23}
3. 1.0×10^{23} and -1.0×10^{23}
4. 1.0×10^{23} and -1.0×10^{-23}

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932072
Option 1 ID : 3398938057
Option 2 ID : 3398938058
Option 3 ID : 3398938059
Option 4 ID : 3398938060
Status : Answered
Chosen Option : 2

Q.18 An ant finds a sugar heap having 1000 grains. It takes one grain back to the anthill and then takes a friend along, to fetch one grain back each. Ants repeat the trips, each ant adding a new friend at every trip, until the heap is exhausted. The number of ants that went out is

1. 256
2. 450
3. 512
4. 1000

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932062
Option 1 ID : 3398938017
Option 2 ID : 3398938018
Option 3 ID : 3398938019
Option 4 ID : 3398938020
Status : Not Answered
Chosen Option : --

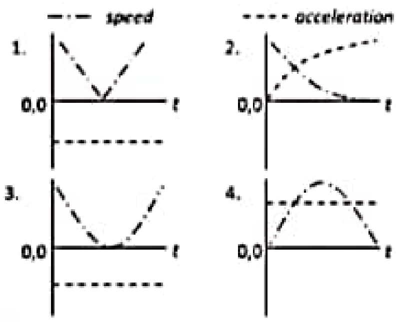
Q.19 The product of the ten numbers 10, 11, 12, 19, 20, 23, 25, 33, 35, and 50 is divisible by 10^x. What is the largest possible value of x?

1. 4
2. 5
3. 6
4. 7

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **3398932071**
Option 1 ID : **3398938053**
Option 2 ID : **3398938054**
Option 3 ID : **3398938055**
Option 4 ID : **3398938056**
Status : **Answered**
Chosen Option : **2**

Q.20 Which of the following graphs shows the speed and the acceleration of a ball thrown up from the Earth and falling back vertically?



Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **3398932076**
Option 1 ID : **3398938073**
Option 2 ID : **3398938074**
Option 3 ID : **3398938075**
Option 4 ID : **3398938076**
Status : **Not Answered**
Chosen Option : --

Section : **Part B Mathematical Sciences**

Q.1 Which of the following is the Jordan canonical form of $\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$ over $\mathbb{R}$?

1. $\begin{bmatrix}0&0&0\\0&0&0\\0&0&3\end{bmatrix}$
2. $\begin{bmatrix}0&1&0\\0&0&0\\0&0&3\end{bmatrix}$
3. $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$
4. $\begin{bmatrix}0&1&0\\0&0&1\\0&0&3\end{bmatrix}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **3398932088**
Option 1 ID : **3398938121**
Option 2 ID : **3398938122**
Option 3 ID : **3398938123**
Option 4 ID : **3398938124**
Status : **Answered**
Chosen Option : **2**

Q.2 Let A and B be 3×3 matrices with real entries.

Let $V_1 = \{v \in \mathbb{R}^3 \mid ABv = 0\}$,

$V_2 = \{v \in \mathbb{R}^3 \mid Bv = 0\}$, and

$V_3 = \{v \in \mathbb{R}^3 \mid Av = 0\}$.

Which of the following is necessarily true?

1. $\dim V_1 = \dim V_2 \Rightarrow A$ is invertible
2. $\dim V_1 = \dim V_3 \Rightarrow A$ is invertible
3. A is invertible $\Rightarrow \dim V_1 = \dim V_2$
4. A is invertible $\Rightarrow \dim V_1 = \dim V_3$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932085
Option 1 ID : 3398938109
Option 2 ID : 3398938110
Option 3 ID : 3398938111
Option 4 ID : 3398938112
Status : Not Answered
Chosen Option : --

Q.3 Let $\mathbb{R}$ denote the set of real numbers and X be a non-empty set. Let Y be the set of all functions from $\mathbb{R}$ to X. Then which of the following is true?

1. If X is finite, then Y is countable
2. Y is always infinite
3. If Y is infinite, then Y is uncountable
4. If Y is uncountable, then X is uncountable

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932078
Option 1 ID : 3398938081
Option 2 ID : 3398938082
Option 3 ID : 3398938083
Option 4 ID : 3398938084
Status : Answered
Chosen Option : 2

Q.4 Consider the ordered basis $v_1 = (1,0,0)$, $v_2 = (1,1,0)$, $v_3 = (1,1,1)$ of $\mathbb{R}^3$. What are the co-ordinates of $(1,2,3)$ with respect to this basis?

1. $(-1,1,3)$
2. $(-1,-1,3)$
3. $(1,1,3)$
4. $(1,-1,3)$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932084
Option 1 ID : 3398938105
Option 2 ID : 3398938106
Option 3 ID : 3398938107
Option 4 ID : 3398938108
Status : Answered
Chosen Option : 2

Q.5 Let $f: X \to Y$ be a function and $(A_n)_{n\geq 1}$ be a sequence of subsets of X. For $A \subset X, A \neq \emptyset$, let $f(A) = \{f(a): a \in A\}$ and $f(\emptyset) = \emptyset$. Then which of the following is true?

1. $f\left(\bigcap_{n\geq 1} A_n\right) = \bigcap_{n\geq 1} f(A_n)$
2. $f\left(\bigcap_{n\geq 1} A_n\right)$ is a proper subset of $\bigcap_{n\geq 1} f(A_n)$
3. $f\left(\bigcup_{n\geq 1} A_n\right) = \bigcup_{n\geq 1} f(A_n)$
4. For any nonempty proper subset A of X, $f(A^c) = f(A)^c$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932080
Option 1 ID : 3398938089
Option 2 ID : 3398938090
Option 3 ID : 3398938091
Option 4 ID : 3398938092
Status : Not Answered
Chosen Option : --

Q.6 Let A and B be invertible $n \times n$ matrices with entries in $\mathbb{R}$ such that AB is diagonalizable with eigenvalues $\lambda_1, \ldots, \lambda_n$. Which of the following is NOT always true?

1. BA is invertible
2. BA is diagonalizable
3. $BA = AB$
4. Eigenvalues of BA are $\lambda_1, \ldots, \lambda_n$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932086
Option 1 ID : 3398938113
Option 2 ID : 3398938114
Option 3 ID : 3398938115
Option 4 ID : 3398938116
Status : Not Answered
Chosen Option : --

Q.7 Let $\alpha > 0$ be a real number. Then $\lim_{n\to\infty} \frac{1}{n^{\alpha+1}}(1^\alpha + 2^\alpha + \cdots + n^\alpha)$ is

1. ∞
2. equal to 0
3. equal to 1
4. positive and strictly less than 1

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932082
Option 1 ID : 3398938097
Option 2 ID : 3398938098
Option 3 ID : 3398938099
Option 4 ID : 3398938100
Status : Answered
Chosen Option : 3

Q.8 Let $A = \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$ be a matrix. Which of the following is true?

1. $A^* = A^{-1}$
2. $AA^* = A^*A$
3. $A^* = A$
4. $A^2 = Id$ (2×2 identity matrix)

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **3398932087**
Option 1 ID : **3398938117**
Option 2 ID : **3398938118**
Option 3 ID : **3398938119**
Option 4 ID : **3398938120**
Status : **Answered**
Chosen Option : **2**

Q.9 Let $f(t) = t^n + a_{n-1}t^{n-1} + \cdots + a_1 t + a_0$ be a non-constant even degree polynomial with real coefficients and $a_0 < 0$. Then the polynomial f

1. need not have a real root.
2. has at least two distinct real roots
3. has at least two real roots but need not be distinct
4. can have at most one real root

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **3398932083**
Option 1 ID : **3398938101**
Option 2 ID : **3398938102**
Option 3 ID : **3398938103**
Option 4 ID : **3398938104**
Status : **Answered**
Chosen Option : **2**

Q.10 Let $x > 100$ be a given real number, which is not an integer. Let S be the set of all rational numbers $r \le x$ of the form

$$r = [x] + \frac{a_1}{10} + \frac{a_2}{10^2} + \cdots + \frac{a_n}{10^n}$$

for some natural number $n \ge 1$, where $a_1, \ldots, a_n$ are integers such that $0 \le a_i \le 9$ and $[x]$ denotes the largest integer $\le x$. Then which one of the following is true?

1. S is infinite and the supremum of S is x
2. S is infinite and the supremum of S is an integer
3. S is finite and the supremum of S is x
4. S is finite and the supremum of S is an integer

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **3398932079**
Option 1 ID : **3398938085**
Option 2 ID : **3398938086**
Option 3 ID : **3398938087**
Option 4 ID : **3398938088**
Status : **Answered**
Chosen Option : **1**

Q.11 Consider a quadratic form $Q(x,y,z)$ on $\mathbb{R}^3$. Let $E_Q = \{(x,y,z): Q(x,y,z) = 1\}$. For which of the following is E_Q a bounded subset of $\mathbb{R}^3$?

1. $Q(x,y,z) = x^2 + y^2 + z^2$
2. $Q(x,y,z) = x^2 + y^2 - z^2$
3. $Q(x,y,z) = x^2 - y^2 - z^2$
4. $Q(x,y,z) = -x^2 + y^2 - z^2$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932089
Option 1 ID : 3398938125
Option 2 ID : 3398938126
Option 3 ID : 3398938127
Option 4 ID : 3398938128
Status : Not Answered
Chosen Option : --

Q.12 Let $f: \mathbb{R} \to \mathbb{R}$ be a continuous function and $(x_n)_{n\geq 1}$ be a bounded sequence of real numbers. Then which of the following is true?

1. $\liminf_{n\to\infty} f(x_n) = f\left(\liminf_{n\to\infty} x_n\right)$
2. $\limsup_{n\to\infty} f(x_n) = f\left(\limsup_{n\to\infty} x_n\right)$
3. $\liminf_{n\to\infty} f(x_n) \leq \limsup_{n\to\infty} x_n$
4. $\liminf_{n\to\infty} f(x_n) \leq f\left(\liminf_{n\to\infty} x_n\right)$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932081
Option 1 ID : 3398938093
Option 2 ID : 3398938094
Option 3 ID : 3398938095
Option 4 ID : 3398938096
Status : Not Answered
Chosen Option : --

Q.13 Let $\alpha \in \mathbb{R}$. Consider $\mathbb{R}$ and $\mathbb{R}^2$ with the usual topology. Consider the set $A = \{(x,y) \in \mathbb{R}^2 / xy = 1\} \cup \{(0,0)\}$ in $\mathbb{R}^2$ with subspace topology. The maps $f_1: A \to \mathbb{R}$ and $f_2: A \to \mathbb{R}$ are given by $f_1(x,y) = x + \alpha$ and $f_2(x,y) = y$. Which of the following statements is true?

1. The map f_2 is continuous but f_1 is not continuous
2. The maps f_1 and f_2 are open maps
3. The maps f_1 and f_2 are closed maps
4. The set $\{(x,y) \in A \mid f_1(x,y) = f_2(x,y)\}$ is closed

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932097
Option 1 ID : 3398938157
Option 2 ID : 3398938158
Option 3 ID : 3398938159
Option 4 ID : 3398938160
Status : Not Answered
Chosen Option : --

Q.14 Let S_5 denote the symmetric group on 5 letters. Which of the following is always true?

1. If $\sigma \in S_5$ is a k-cycle, then σ^i is also a k-cycle for all $2 \leq k \leq 5$ and $i \geq 1$.
2. If $\sigma, \tau \in S_5$ have both order 5, then $\sigma\tau$ also has order 5
3. Any 3-cycle in S_5 can be written as a product of two 5-cycles
4. Any 2-cycle in S_5 can be written as a product of two 5-cycles

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **3398932095**
Option 1 ID : **3398938149**
Option 2 ID : **3398938150**
Option 3 ID : **3398938151**
Option 4 ID : **3398938152**
Status : **Not Answered**
Chosen Option : --

Q.15 Let R_1 be the ring $\mathbb{Z}/(11\,\mathbb{Z})$ and let R_2 be the ring $\mathbb{Z}/(13\,\mathbb{Z})$. Let

A = number of ideals in the product ring $R_1 \times R_2$

B = number of ring homomorphisms $R_1 \to R_2$ sending 1 to 1

C = number of ring homomorphisms $R_2 \to R_1$ sending 1 to 1

What is $A + B + C$?

1. 3
2. 4
3. 6
4. 8

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **3398932094**
Option 1 ID : **3398938145**
Option 2 ID : **3398938146**
Option 3 ID : **3398938147**
Option 4 ID : **3398938148**
Status : **Not Answered**
Chosen Option : --

Q.16 Let F be a finite field of order q. What is the number of 2-dimensional subspaces of the vector space F^3 over F?

1. q^3
2. q^2
3. $(q^3 - 1)/(q - 1)$
4. $q^3 - 1$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **3398932096**
Option 1 ID : **3398938153**
Option 2 ID : **3398938154**
Option 3 ID : **3398938155**
Option 4 ID : **3398938156**
Status : **Not Answered**
Chosen Option : --

Q.17 Which of the following sets lies in the region of convergence of $\sum_{n=0}^{\infty}(3z - 2i)^{3n}$?

1. $\left\{x + \frac{2i}{3} : x \in \left(\frac{-1}{3}, \frac{1}{3}\right)\right\}$
2. $\left\{x + \frac{2i}{3} : x \in \left(\frac{-1}{2}, \frac{1}{9}\right)\right\}$

3. $\left\{x + \frac{2i}{3} : x \in \left(\frac{-1}{27}, \frac{20}{27}\right)\right\}$

4. $\left\{x + \frac{2i}{3} : x \in \left(\frac{-5}{6}, \frac{5}{6}\right)\right\}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **3398932092**
Option 1 ID : **3398938137**
Option 2 ID : **3398938138**
Option 3 ID : **3398938139**
Option 4 ID : **3398938140**
Status : **Not Answered**
Chosen Option : **--**

Q.18 Let f be a non-constant entire function and γ the positively oriented circle $\{z \in \mathbb{C} : |z| = 2\}$. Then

$$\frac{1}{\pi}\int_{\gamma} \frac{f(z)}{z^2+1}\,dz \text{ equals}$$

1. $f(1) + f(-1)$
2. $f(1) - f(-1)$
3. $f(i) + f(-i)$
4. $f(i) - f(-i)$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **3398932090**
Option 1 ID : **3398938129**
Option 2 ID : **3398938130**
Option 3 ID : **3398938131**
Option 4 ID : **3398938132**
Status : **Answered**
Chosen Option : **4**

Q.19 Let $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ be the open unit disc and $f : \mathbb{D} \to \mathbb{C}$ a holomorphic function such that $|f(z)| \le 1$ on $\mathbb{D}$. Suppose that $f(0) = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$. Then

1. $f\left(-\frac{1}{2}\right) = \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$
2. $f\left(-\frac{1}{2}\right) = \frac{1}{\sqrt{3}} - \frac{i}{\sqrt{3}}$
3. $f\left(\frac{1}{2}\right) = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$
4. $f\left(\frac{1}{2}\right) = \frac{1}{\sqrt{3}} + \frac{i}{\sqrt{3}}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **3398932093**
Option 1 ID : **3398938141**
Option 2 ID : **3398938142**
Option 3 ID : **3398938143**
Option 4 ID : **3398938144**
Status : **Not Answered**
Chosen Option : **--**

Q.20 For which value of a among the following does $f(z) = \frac{az+2}{3z+1}$ map the upper half plane $\mathbb{H} = \{z \in \mathbb{C} : z = x + iy, y > 0\}$ to $\mathbb{H}$?

1. 1
2. 3

3. 3
4. 4

Question Type : **MCQ**
Question ID : **3398932091**
Option 1 ID : **3398938133**
Option 2 ID : **3398938134**
Option 3 ID : **3398938135**
Option 4 ID : **3398938136**
Status : **Answered**
Chosen Option : **4**

Q.21 Let $\bar{q} = (q_1, \dots, q_n)$ be the generalized coordinates and $\bar{p} = (p_1, \dots, p_n)$ be the generalized momenta of a system. Let the Poisson bracket of two quantities f, g be denoted by $[f, g]_{\bar{q},\bar{p}}$ and the Hamiltonian of the system be H. Then which of the following statements is true?

1. If $\frac{\partial f}{\partial t} = 0, [H, f]_{\bar{q},\bar{p}} = 0$ then f is a conserved quantity
2. If f is a conserved quantity then $[f, p_i]_{\bar{q},\bar{p}} = 0,\ 1 \le i \le n$
3. If f is a conserved quantity then $[f, q_i]_{\bar{q},\bar{p}} = 0,\ 1 \le i \le n$
4. There exists a canonical transformation $(\bar{q}, \bar{p}) \to (\bar{Q}, \bar{P})$ such that $[f, g]_{\bar{q},\bar{p}} \ne [f, g]_{\bar{Q},\bar{P}}$ for some f, g

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **3398932105**
Option 1 ID : **3398938189**
Option 2 ID : **3398938190**
Option 3 ID : **3398938191**
Option 4 ID : **3398938192**
Status : **Not Answered**
Chosen Option : **--**

Q.22 Assume that $a, b \in \mathbb{R}\setminus\{0\}$ and $a^2 \ne b^2$. Suppose that the Gauss-Seidel method is used to solve the system of equations

$$\begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Then the set of all values of (a, b) such that the method converges for every choice of initial vector is

1. $\{(a,b) \mid a^2 < b^2\}$
2. $\{(a,b) \mid a < |b|\}$
3. $\{(a,b) \mid |b| < |a|\}$
4. $\{(a,b) \mid a^2 + b^2 < 1\}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **3398932102**
Option 1 ID : **3398938177**
Option 2 ID : **3398938178**
Option 3 ID : **3398938179**
Option 4 ID : **3398938180**
Status : **Not Answered**
Chosen Option : **--**

Q.23 Let $u(x,y,z) = C_1$ and $v(x,y,z) = C_2$ be two independent families of smooth surfaces. Let $P, Q, R \in C^1(\mathbb{R}^3)$ and for any $\xi \in \mathbb{R}^3$, that lies on the curve of intersection of $u(x,y,z) = C_1$ and $v(x,y,z) = C_2$, $(P(\xi), Q(\xi), R(\xi))$ is in the direction of the tangent to the curve of intersection at ξ. Then a general solution z of $P\frac{\partial z}{\partial x} + Q\frac{\partial z}{\partial y} = R$ is implicitly given by

1. $u + v = f(u + v)$ for every $f \in C^1(\mathbb{R})$

2. $uv = f(u)$ for every $f \in C^1(\mathbb{R})$

3. $u = f(uv)$ for every $f \in C^1(\mathbb{R})$

4. $uv = f(uv)$ for every $f \in C^1(\mathbb{R})$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932101
Option 1 ID : 3398938173
Option 2 ID : 3398938174
Option 3 ID : 3398938175
Option 4 ID : 3398938176
Status : Not Answered
Chosen Option : --

Q.24 The critical point (0,0) for the system of equations

$$x'(t) = x^2 + y^2 - 2x$$
$$y'(t) = 3x^2 - x + 3y$$

is a

1. stable point

2. source point

3. saddle point

4. spiral stable point

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932099
Option 1 ID : 3398938165
Option 2 ID : 3398938166
Option 3 ID : 3398938167
Option 4 ID : 3398938168
Status : Not Answered
Chosen Option : --

Q.25 The extremal for the following functional

$$\int_0^1 \left(\alpha t x(t) + \left(x'(t)\right)^2\right) dt, \qquad \alpha \neq 0,$$

where $x(0) = 1, x'(0) = 0$, is

1. $\frac{\alpha}{12} t^3 + 1$

2. $t^3 + \alpha t^2 + 1$

3. $t^2 + t + 1$

4. $t^4 + \alpha t^2 + 1$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932103
Option 1 ID : 3398938181
Option 2 ID : 3398938182
Option 3 ID : 3398938183
Option 4 ID : 3398938184
Status : Answered
Chosen Option : 1

Q.26 The 2nd order partial differential equation

$$e^x \frac{\partial^2 u}{\partial x^2} + (x^2 + y^2 - 1)\frac{\partial^2 u}{\partial y^2} = 0 \text{ is}$$

1. elliptic inside the unit circle with centre at (0,0)
2. hyperbolic outside the unit circle with centre at (0,0)
3. elliptic outside the unit circle with centre at (0,0)
4. parabolic for $x, y \in \mathbb{R}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **3398932100**
Option 1 ID : **3398938169**
Option 2 ID : **3398938170**
Option 3 ID : **3398938171**
Option 4 ID : **3398938172**
Status : **Answered**
Chosen Option : **3**

Q.27 Let $|f(x)| \leq 4|x|$ for $x \in \mathbb{R}$. The largest possible value of $|x(1)|$ where $x'(t) = f(x(t)), t > 0, x(0) = 3,$ is

1. $3e^4$
2. $4e^3$
3. $12e^3$
4. $12e^4$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **3398932098**
Option 1 ID : **3398938161**
Option 2 ID : **3398938162**
Option 3 ID : **3398938163**
Option 4 ID : **3398938164**
Status : **Answered**
Chosen Option : **1**

Q.28 If $\varphi: [0,1] \to \mathbb{R}$ is continuous and satisfies

$$\int_0^x (x-t)^2 \varphi(t)dt = x^4 + x^5,$$

then $\varphi(1) =$

1. 18
2. 30
3. 42
4. 48

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **3398932104**
Option 1 ID : **3398938185**
Option 2 ID : **3398938186**
Option 3 ID : **3398938187**
Option 4 ID : **3398938188**
Status : **Not Answered**
Chosen Option : **--**

Q.29 The goal is to estimate the unknown parameter μ as accurately as possible. You are offered a choice between two data sets. Assume $n \geq 2$.

(A) : $X_1, X_2, \ldots, X_n$ i.i.d. normal with mean μ and variance 1

(B): $Y_1, Y_2, \ldots, Y_n$ i.i.d. normal with mean 2μ and variance 2

Then

1. data (A) is preferable

2. data (B) is preferable

3. both data sets are equally good

4. which data set is preferable depends on the value of n

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932117
Option 1 ID : 3398938237
Option 2 ID : 3398938238
Option 3 ID : 3398938239
Option 4 ID : 3398938240
Status : Not Answered
Chosen Option : --

Q.30 In a simple linear regression model, assume that the random errors $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n$ are uncorrelated and homoscedastic. Then we may conclude that the residuals $e_1, e_2, \ldots, e_n$ are

1. uncorrelated and homoscedastic

2. correlated and homoscedastic

3. uncorrelated and heteroscedastic

4. correlated and heteroscedastic

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932113
Option 1 ID : 3398938221
Option 2 ID : 3398938222
Option 3 ID : 3398938223
Option 4 ID : 3398938224
Status : Not Answered
Chosen Option : --

Q.31 Consider the set Ω of all 4-tuples (x_1, x_2, x_3, x_4) of integers such that each $x_i \geq 0$ and $x_1 + x_2 + x_3 + x_4 = 90$. If a point is selected uniformly at random from Ω, the conditional probability that $x_1 \geq 1$ given that $x_3 \geq 44$ and $x_4 \geq 44$ equals

1. $\frac{1}{4}$

2. $\frac{1}{3}$

3. $\frac{2}{5}$

4. $\frac{1}{8}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932106
Option 1 ID : 3398938193
Option 2 ID : 3398938194
Option 3 ID : 3398938195
Option 4 ID : 3398938196
Status : Not Answered
Chosen Option : --

Q.32 Let $X_1, X_2, \ldots, X_n$, be a random sample from $N(\theta, 1)$ distribution. For testing $H_0: \theta = 0$ against $H_1: \theta < 0$, if the UMP test is used and if the observed sample mean is 1, then

1. the test is biased

2. H_0 is rejected at 5% level of significance

3. H_0 is **NOT** rejected at 1% level of significance

4. p-value of the test is less than $\frac{1}{2}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **3398932111**
Option 1 ID : **3398938213**
Option 2 ID : **3398938214**
Option 3 ID : **3398938215**
Option 4 ID : **3398938216**
Status : **Not Answered**
Chosen Option : --

Q.33 Suppose $B = ((b_{ij})) \sim W_p(k, \Sigma)$ (Wishart distribution), where $p = 7,\ k = 12$ and $\Sigma = ((\sigma_{ij}))$ is a positive definite matrix. Then the distribution of $\frac{\sum_{i=1}^{p}\sum_{j=1}^{p} b_{ij}}{\sum_{i=1}^{p}\sum_{j=1}^{p} \sigma_{ij}}$ is

1. χ_1^2

2. χ_7^2

3. χ_{12}^2

4. χ_5^2

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **3398932114**
Option 1 ID : **3398938225**
Option 2 ID : **3398938226**
Option 3 ID : **3398938227**
Option 4 ID : **3398938228**
Status : **Not Answered**
Chosen Option : --

Q.34 Consider a Markov chain with state space $S = \{0,1,\dots,1000\}$ and transition probabilities given by $p_{i,i+1} = 1$ for $0 \le i \le 999$ and $p_{1000,1000} = p_{1000,0} = \frac{1}{2}$. Then

1. $\lim_{n\to\infty} p_{ij}^{(n)} = \frac{1}{1000}$ for all i,j

2. $\lim_{n\to\infty} p_{ij}^{(n)} = \frac{1}{1000}$ for all $i,j \le 999$

3. $\lim_{n\to\infty} p_{ij}^{(n)} = \frac{1}{1001}$ for all i,j

4. $\lim_{n\to\infty} p_{ij}^{(n)} = \frac{1}{1002}$ for all $i,j \le 999$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **3398932108**
Option 1 ID : **3398938201**
Option 2 ID : **3398938202**
Option 3 ID : **3398938203**
Option 4 ID : **3398938204**
Status : **Not Answered**
Chosen Option : --

Q.35 Let $X_1, X_2, \dots, X_n$ be i.i.d. Uniform$(0,\theta)$ random variables where $\theta > 0$. Then, a consistent estimator of θ is

1. $\min(X_1,\dots,X_n) + \frac{1}{n}$

2. $\frac{1}{n}(X_1 + \cdots + X_n)$

3. $\max(X_1, \dots, X_n) - \frac{1}{n}$

4. $\frac{n}{X_1 + \cdots + X_n}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932110
Option 1 ID : 3398938209
Option 2 ID : 3398938210
Option 3 ID : 3398938211
Option 4 ID : 3398938212
Status : Not Answered
Chosen Option : --

Q.36 In a block design with b blocks and v treatments, every treatment pair occurs exactly once in each block. Then the resulting design will be

1. connected and incomplete
2. not connected and complete
3. connected and complete
4. not connected and incomplete

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932116
Option 1 ID : 3398938233
Option 2 ID : 3398938234
Option 3 ID : 3398938235
Option 4 ID : 3398938236
Status : Not Answered
Chosen Option : --

Q.37 From a population of $N = nk$ units, a sample of size n is drawn using linear systematic sampling scheme. Then the inclusion probability of the i^{th} unit for $i = 1, \dots, N$ is

1. $\frac{1}{N}$
2. $\frac{1}{n}$
3. $\frac{1}{k}$
4. $\frac{k!}{N!}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ
Question ID : 3398932115
Option 1 ID : 3398938229
Option 2 ID : 3398938230
Option 3 ID : 3398938231
Option 4 ID : 3398938232
Status : Not Answered
Chosen Option : --

Q.38 Let $N, X_1, X_2, X_3, \dots$ be independent random variables where N has Poisson (λ) distribution and X_k has normal distribution with mean k and variance k^2 for each $k \geq 1$. Then, the variance of X_{N+1} is

1. λ
2. $\lambda^2 + 3\lambda + 1$
3. $\lambda^2 + 4\lambda + 2$
4. $\lambda^2 + 4\lambda + 1$

4. $x^2 + 4x + 1$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : **MCQ**
Question ID : **3398932107**
Option 1 ID : **3398938197**
Option 2 ID : **3398938198**
Option 3 ID : **3398938199**
Option 4 ID : **3398938200**
Status : **Not Answered**
Chosen Option : **--**

Q.39 X is a Poisson random variable with parameter 20. The conditional distribution of Y given $X = k$ is $\text{Binomial}\left(k, \frac{1}{2}\right)$ for $k \geq 1$ and $P(Y = 0|X = 0) = 1$. Then the distribution (unconditional) of Y is

1. $\text{Poisson}(10)$
2. $\text{Poisson}(20)$
3. $\text{Poisson}(40)$
4. $\text{Binomial}\left(20, \frac{1}{2}\right)$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : **MCQ**
Question ID : **3398932109**
Option 1 ID : **3398938205**
Option 2 ID : **3398938206**
Option 3 ID : **3398938207**
Option 4 ID : **3398938208**
Status : **Not Answered**
Chosen Option : **--**

Q.40 Let X_1, X_2 be i.i.d. random variables from an exponential distribution with mean $\frac{1}{\theta}$ where $\theta > 0$. Suppose that the prior distribution for θ is exponential with mean 1. Then the Bayes estimator for θ with respect to the squared error loss function is

1. $\frac{X_1 + X_2}{2} + 1$
2. $\frac{2}{X_1 + X_2}$
3. $\frac{3}{X_1 + X_2 + 1}$
4. $\frac{1}{3(X_1 + X_2)}$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : **MCQ**
Question ID : **3398932112**
Option 1 ID : **3398938217**
Option 2 ID : **3398938218**
Option 3 ID : **3398938219**
Option 4 ID : **3398938220**
Status : **Not Answered**
Chosen Option : **--**

Q.1 Let $P(x)$ be a polynomial with real coefficients and of degree $n \geq 1$. Then which of the following are true?

1. If n is odd, then $P(x)$ has a real root
2. If $P(x)$ has only real roots then its derivative $P'(x)$ has only real roots
3. If $P'(x)$ has only real roots, then $P(x)$ has only real roots
4. If n is odd, then $P(x)$ takes every real value as x varies over reals

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932119
Option 1 ID : 3398938245
Option 2 ID : 3398938246
Option 3 ID : 3398938247
Option 4 ID : 3398938248
Status : Answered
Chosen Option : 1,2,4

Q.2 Let $f: [a, b) \to \mathbb{R}$ be a continuous function. Define $F: [a, b) \to \mathbb{R}$ by
$F(x) = \max_{t \in [a,x]} f(t)$

Then which of the following are true?

1. F need not be continuous
2. F is necessarily monotonic
3. F is necessarily bounded
4. F is Riemann integrable on $[a, x]$ for every $x \in [a, b)$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932122
Option 1 ID : 3398938257
Option 2 ID : 3398938258
Option 3 ID : 3398938259
Option 4 ID : 3398938260
Status : Answered
Chosen Option : 1,2,3

Q.3 Let v_1, v_2, w_1, w_2 be vectors in $\mathbb{R}^4$ such that $v_1, v_2, v_1 + w_1, v_2 + w_2$ is a basis of $\mathbb{R}^4$. Which of the following are bases of $\mathbb{R}^4$?

1. $v_1, v_2, 2v_1 + 3w_2, 5v_2 - 3w_1$
2. $v_1, v_2, 2v_1 + 2w_1, v_1 + w_2$
3. $v_1 + v_2, v_2, w_1, 2v_1 + 5v_2 - 4w_1$
4. $v_1 + v_2, v_2, w_1 + w_2, 2v_1 + v_2 + w_1 + w_2$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932130
Option 1 ID : 3398938289
Option 2 ID : 3398938290
Option 3 ID : 3398938291
Option 4 ID : 3398938292
Status : Answered
Chosen Option : 1,2

Q.4 Let (x_n) be a sequence of real numbers with $|x_n| > 2$. Then which of the following are true?

1. $\lim_{n\to\infty} x_n = \infty \Rightarrow \lim_{n\to\infty} \left(1 + \frac{1}{x_n}\right)^{x_n} = \infty$
2. $\lim_{n\to\infty} x_n = \infty \Rightarrow \lim_{n\to\infty} \left(1 + \frac{1}{x_n}\right)^{x_n} = 1$
3. $\lim_{n\to\infty} x_n = -\infty \Rightarrow \lim_{n\to\infty} \left(1 + \frac{1}{x_n}\right)^{x_n} = 0$

4. $\lim_{n\to\infty} x_n = -\infty \Rightarrow \lim_{n\to\infty}\left(1+\frac{1}{x_n}\right)^{x_n} > 1$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932118
Option 1 ID : 3398938241
Option 2 ID : 3398938242
Option 3 ID : 3398938243
Option 4 ID : 3398938244
Status : Answered
Chosen Option : 4

Q.5 For $x, y \in \mathbb{R}^2$ define $\langle x, y\rangle = x^t A y$, where A is a 2×2 matrix with real entries. For which of the following choices of A does this define an inner product on $\mathbb{R}^2$?

1. $A = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$
2. $A = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$
3. $A = \begin{bmatrix} 2 & 2 \\ 2 & 1 \end{bmatrix}$
4. $A = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932133
Option 1 ID : 3398938301
Option 2 ID : 3398938302
Option 3 ID : 3398938303
Option 4 ID : 3398938304
Status : Answered
Chosen Option : 1,4

Q.6 Let A be a 3×3 matrix over $\mathbb{C}$. Let $\omega = e^{2\pi i/3}$. Which of the following are true?

1. If $A^3 = I$, then the eigenvalues of A are $1, \omega, \omega^2$
2. If $A^3 = I$, then every eigenvalue of A is in the set $\{1, \omega, \omega^2\}$
3. If the eigenvalues of A are $1, \omega, \omega^2$, then $A^3 = I$
4. If every eigenvalue of A is in the set $\{1, \omega, \omega^2\}$, then $A^3 = I$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932132
Option 1 ID : 3398938297
Option 2 ID : 3398938298
Option 3 ID : 3398938299
Option 4 ID : 3398938300
Status : Answered
Chosen Option : 1,2,3,4

Q.7 Let I be any interval in $\mathbb{R}$. Let $f, g: I \to \mathbb{R}$ be uniformly continuous functions. If $h(x) = f(x)g(x)$, then h is uniformly continuous if

1. either f or g is bounded
2. I is compact
3. I is bounded
4. I is closed

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932121
Option 1 ID : 3398938253
Option 2 ID : 3398938254
Option 3 ID : 3398938255
Option 4 ID : 3398938256
Status : Answered
Chosen Option : 2,3,4

Q.8 Consider the infinite series

$$\sum_{n=1}^{\infty}(-1)^n \frac{\cos(nx)}{n^2+x^2}, \quad \text{where } x \in \mathbb{R}.$$

The largest set on which it converges uniformly is:

1. $\mathbb{R}$
2. $\mathbb{R}\setminus\{0\}$
3. $\mathbb{R}\setminus\{n\pi : n \in \mathbb{Z}\}$
4. $[0, 2\pi]$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932120
Option 1 ID : 3398938249
Option 2 ID : 3398938250
Option 3 ID : 3398938251
Option 4 ID : 3398938252
Status : Not Answered
Chosen Option : --

Q.9 Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined as $f(x,y) = x|x| + |y|$. Then which of the following are true?

1. f is differentiable at $(0,0)$
2. f is not differentiable at $(0,0)$ but all its directional derivatives at $(0,0)$ exist
3. $\frac{\partial f}{\partial x}(0,0)$ exists and equals 0
4. $\frac{\partial f}{\partial y}(0,0)$ exists and equals 0

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932125
Option 1 ID : 3398938273
Option 2 ID : 3398938274
Option 3 ID : 3398938275
Option 4 ID : 3398938276
Status : Answered
Chosen Option : 1,3,4

Q.10 Consider $f_n(x) = \frac{1}{n}e^{-x/n}$ for $x \in [0, \infty)$. Then which of the following are true?

1. $\{f_n\}$ converges uniformly
2. The sequence of derivatives $\{f_n'\}$ converges uniformly
3. $\lim_{n\to\infty} \int_0^{\infty} f_n(x)\,dx \neq \int_0^{\infty} \left(\lim_{n\to\infty} f_n(x)\right) dx$

4. $\lim_{n\to\infty} \int_0^\infty f_n(x)\,dx = \int_0^\infty \left(\lim_{n\to\infty} f_n(x)\right) dx$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **3398932123**
Option 1 ID : **3398938261**
Option 2 ID : **3398938262**
Option 3 ID : **3398938263**
Option 4 ID : **3398938264**
Status : **Answered**
Chosen Option : **2,3**

Q.11 For any interval I of $\mathbb{R}$ let $BV(I)$ denote the space of all functions on I which are of bounded variation. Then which of the following statements are true?

1. If f, g belong to $BV([a,b])$ then the product $f.g$ belongs to $BV([a,b])$
2. If $\{f_n\}$ is a sequence in $BV([a,b])$ that converges to f pointwise, then f belongs to $BV([a,b])$
3. If $f:[0,\infty) \to \mathbb{R}$ is a function such that f restricted to $[n, n+1]$ belongs to $BV([n,n+1])$ for $n = 0,1,2,\ldots$, then f belongs to $BV([0,\infty))$
4. The function $f(x) = \begin{cases} \frac{e^x - 1}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$ belongs to $BV([a,b])$ for any $a < b$.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **3398932124**
Option 1 ID : **3398938265**
Option 2 ID : **3398938266**
Option 3 ID : **3398938267**
Option 4 ID : **3398938268**
Status : **Not Answered**
Chosen Option : **--**

Q.12 Which of the following matrices are diagonalizable over $\mathbb{C}$?

1. $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
2. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$
3. $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
4. $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **3398932128**
Option 1 ID : **3398938281**
Option 2 ID : **3398938282**
Option 3 ID : **3398938283**
Option 4 ID : **3398938284**
Status : **Answered**
Chosen Option : **2,3**

Q.13 Let V be an n dimensional inner product space over $\mathbb{R}$, where $n \geq 2$. A linear map $T: V \to V$ is said to be an orthogonal map if $\langle Tv, w\rangle = \langle v, T^{-1}w\rangle$ for all $v, w \in V$. Which of the following are true?

1. T is orthogonal $\Longrightarrow$ T is diagonalizable over $\mathbb{R}$
2. T diagonalizable over $\mathbb{R} \Longrightarrow T$ is orthogonal
3. T is orthogonal $\Leftrightarrow \langle Tv, Tw\rangle = \langle v, w\rangle$ for all $v, w \in V$
4. T is orthogonal $\Leftrightarrow \langle Tv, Tv\rangle = \langle v, v\rangle$ for all $v \in V$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932134
Option 1 ID : 3398938305
Option 2 ID : 3398938306
Option 3 ID : 3398938307
Option 4 ID : 3398938308
Status : Answered
Chosen Option : 2,3

Q.14 Let T_1, T_2 be linear operators on $\mathbb{R}^2$ such that $T_1T_2 = 0$. Which of the following are necessarily true?

1. 0 is an eigenvalue of T_1 or 0 is an eigenvalue of T_2
2. There exists a nonzero vector $w \in \mathbb{R}^2$ such that $T_1w = T_2w = 0$
3. $T_2T_1 = 0$
4. $T_1 = 0$ or $T_2 = 0$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932131
Option 1 ID : 3398938293
Option 2 ID : 3398938294
Option 3 ID : 3398938295
Option 4 ID : 3398938296
Status : Not Answered
Chosen Option : --

Q.15 Let (X, d) be a metric space. Which of the following are NOT in general a metric on X?

1. $\rho(x,y) = (d(x,y))^2$
2. $\rho(x,y) = \sqrt{d(x,y)}$
3. $\rho(x,y) = \min\{d(x,y), 1\}$
4. $\rho(x,y) = \begin{cases} 0 \text{ if } d(x,y) = 0 \\ 1 \text{ if } d(x,y) > 0 \end{cases}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932127
Option 1 ID : 3398938277
Option 2 ID : 3398938278
Option 3 ID : 3398938279
Option 4 ID : 3398938280
Status : Answered
Chosen Option : 3

Q.16 Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined as

$$f(x,y) = \begin{cases} \dfrac{xy^3}{x^2 + y^6} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

Then which of the following are true?

1. f is bounded on $\mathbb{R}^2$
2. the restriction of f to each line of the form $y = mx$ is continuous at $(0,0)$
3. f is continuous at $(0,0)$
4. f is not continuous at $(0,0)$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **3398932125**
Option 1 ID : **3398938269**
Option 2 ID : **3398938270**
Option 3 ID : **3398938271**
Option 4 ID : **3398938272**
Status : **Not Answered**
Chosen Option : --

Q.17 Let T be a 3×3 matrix with entries in $\mathbb{R}$. Consider the system of linear equations $T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$. Which of the following are true?

1. Rank of $T \leq 2 \implies$ the system has a non-zero solution
2. Rank of $T \leq 1 \implies$ the system has infinitely many solutions
3. Rank of $T \geq 2 \implies$ any two solutions of the system are linearly dependent
4. Rank of $T \geq 1 \implies$ every $\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} \in \mathbf{R}^3$ is a solution of the system

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **3398932135**
Option 1 ID : **3398938309**
Option 2 ID : **3398938310**
Option 3 ID : **3398938311**
Option 4 ID : **3398938312**
Status : **Answered**
Chosen Option : **1,3**

Q.18 Let V be a finite dimensional vector space over a field K. Then, which of the following are true?

1. V is not a union of two proper subspaces
2. If V has exactly two subspaces, then V is isomorphic to K as a vector space
3. V is not a union of finitely many proper subspaces
4. If $\dim V > 1$, then the intersection of all non-zero subspaces of V is a non-zero subspace

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **3398932129**
Option 1 ID : **3398938285**
Option 2 ID : **3398938286**
Option 3 ID : **3398938287**
Option 4 ID : **3398938288**
Status : **Not Answered**
Chosen Option : --

Q.19 Let $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ be the open unit disc in $\mathbb{C}$ and let $f : \mathbb{D} \to \mathbb{D}$ be a non-constant holomorphic function. Suppose that $f(1/3) = 0$. Which of the following are possible values of $\left|f\left(\frac{1}{2}\right)\right|$?

1. $^4/_5$
2. $^3/_5$
3. $^2/_5$
4. $^1/_5$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932139
Option 1 ID : 3398938325
Option 2 ID : 3398938326
Option 3 ID : 3398938327
Option 4 ID : 3398938328
Status : Answered
Chosen Option : 2,3,4

Q.20 Let $I = \langle xy - 1 \rangle \subseteq \mathbb{C}[x, y]$ be an ideal. Which of the following are true?

1. I is a maximal ideal of $\mathbb{C}[x, y]$
2. I is a prime ideal of $\mathbb{C}[x, y]$
3. $\mathbb{C}[x, y]/I$ has infinitely many prime ideals
4. $\mathbb{C}[x, y]/I$ is a finite set

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932144
Option 1 ID : 3398938345
Option 2 ID : 3398938346
Option 3 ID : 3398938347
Option 4 ID : 3398938348
Status : Not Answered
Chosen Option : --

Q.21 How many 3-sylow subgroups does the symmetric group S_4 have?

1. 8
2. 4
3. 2
4. 1

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932143
Option 1 ID : 3398938341
Option 2 ID : 3398938342
Option 3 ID : 3398938343
Option 4 ID : 3398938344
Status : Answered
Chosen Option : 2

Q.22 Let X be a topological space and let A and B be non-empty subsets of X. Suppose $A \cup B$ and $A \cap B$ are connected. Then, which of the following statements are true?

1. At least one of A and B is connected
2. If A is closed and B is open, then both A and B are connected
3. If A and B are open sets, then both A and B are connected
4. If A and B are closed sets, then both A and B are connected

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932146
Option 1 ID : 3398938353
Option 2 ID : 3398938354
Option 3 ID : 3398938355
Option 4 ID : 3398938356
Status : Not Answered
Chosen Option : --

Q.23 Let p, q be prime numbers such that $q-1$ is divisible by p. How many distinct group homomorphisms $f: \mathbb{Z}/p\mathbb{Z} \to (\mathbb{Z}/q\mathbb{Z})^*$ are there? Here $(\mathbb{Z}/q\mathbb{Z})^*$ denotes the group of units in the ring $\mathbb{Z}/q\mathbb{Z}$.

1. 0
2. 1
3. $\frac{q-1}{p}$
4. p

ptions 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932142
Option 1 ID : 3398938337
Option 2 ID : 3398938338
Option 3 ID : 3398938339
Option 4 ID : 3398938340
Status : Answered
Chosen Option : 1

Q.24 Let $\mathbb{D}^* = \{z \in \mathbb{C} : 0 < |z| < 1\}$ be the punctured unit disc in the complex plane. Suppose $f: \mathbb{D}^* \to \mathbb{C}$ is a holomorphic function that satisfies $|f(z)| \le \frac{1}{|z|}$ for $z \in \mathbb{D}^*$. Such an f

1. necessarily extends to a holomorphic function on the unit disc
2. necessarily has an essential singularity at $z = 0$
3. has at most a simple pole at $z = 0$
4. can have a pole of order 2 at $z = 0$

ptions 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932138
Option 1 ID : 3398938321
Option 2 ID : 3398938322
Option 3 ID : 3398938323
Option 4 ID : 3398938324
Status : Not Answered
Chosen Option : --

Q.25 Let f be a holomorphic function on the open unit disc $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ such that $f\left(\frac{1}{n}\right) = \frac{2n+1}{2n-1}$ for $n = 1, 2, \ldots$. Let $\Im(w)$ denote the imaginary part of a complex number w. Then, which of the following are true?

1. $\Im(f(i/2)) > 0$ and $\Im(f(-i/2)) < 0$
2. $\Im(f(i/2)) < 0$ and $\Im(f(-i/2)) > 0$
3. $\Im(f(i/2)) > 0$ and $\Im(f(-i/2)) > 0$
4. $\Im(f(i/2)) < 0$ and $\Im(f(-i/2)) < 0$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ
Question ID : 3398932137
Option 1 ID : 3398938317
Option 2 ID : 3398938318
Option 3 ID : 3398938319
Option 4 ID : 3398938320
Status : Not Answered
Chosen Option : --

Q.26 Which of the following statements are correct?

1. The fields $\mathbb{Q}(\sqrt{5})$ and $\mathbb{Q}(\sqrt{7})$ are isomorphic as vector spaces over $\mathbb{Q}$
2. The fields $\mathbb{Q}(\sqrt{5})$ and $\mathbb{Q}(\sqrt{7})$ are isomorphic as fields
3. The Galois group of $\mathbb{Q}(\sqrt{5})/\mathbb{Q}$ is isomorphic to the Galois group of $\mathbb{Q}(\sqrt{7})/\mathbb{Q}$
4. The fields $\mathbb{Q}(\sqrt{5}, \sqrt{7})$ and $\mathbb{Q}(\sqrt{5}+\sqrt{7})$ are isomorphic as fields

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ
Question ID : 3398932145
Option 1 ID : 3398938349
Option 2 ID : 3398938350
Option 3 ID : 3398938351
Option 4 ID : 3398938352
Status : Not Answered
Chosen Option : --

Q.27 Let $\mathbb{R}^*$ be the multiplicative group of nonzero real numbers. Which of the following statements are true?

1. If $H \subseteq \mathbb{R}^*$ is a subgroup such that $x^2 \in H$ for all $x \in \mathbb{R}^*$, then $H = \mathbb{R}^*$
2. If $H \subseteq \mathbb{R}^*$ is a subgroup such that $x^3 \in H$ for all $x \in \mathbb{R}^*$, then $H = \mathbb{R}^*$
3. There is no subgroup of $\mathbb{R}^*$ of index 2
4. There is no subgroup of $\mathbb{R}^*$ of index 3

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ
Question ID : 3398932140
Option 1 ID : 3398938329
Option 2 ID : 3398938330
Option 3 ID : 3398938331
Option 4 ID : 3398938332
Status : Not Answered
Chosen Option : --

Q.28 The number of $(x_1, x_2, x_3) \in \mathbb{Z}^3$ such that $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$ and such that $x_1 + x_2 + x_3 = 9$ is

1. 55
2. odd
3. $\binom{9}{3}$
4. divisible by 3

Options 1. 1

2. 2

3. 3

4. 4

Question Type : **MSQ**
Question ID : **3398932141**
Option 1 ID : **3398938333**
Option 2 ID : **3398938334**
Option 3 ID : **3398938335**
Option 4 ID : **3398938336**
Status : **Answered**
Chosen Option : **1,2**

Q.29 Let γ be the positively oriented unit circle in the complex plane. The line integral

$$\oint_\gamma e^{2\bar{z}} dz$$

equals

1. 0
2. 1
3. $4\pi i$
4. $-2\pi i$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **3398932136**
Option 1 ID : **3398938313**
Option 2 ID : **3398938314**
Option 3 ID : **3398938315**
Option 4 ID : **3398938316**
Status : **Answered**
Chosen Option : **1**

Q.30 Let $K \subset \mathbb{R}^2$ be an arbitrary non-empty set and let $\pi_1(x,y) = x$ and $\pi_2(x,y) = y$ be the projections from $\mathbb{R}^2$ onto the first and second coordinates respectively. Which of the following statements are true?

1. If K is compact, then $\pi_1(K)$ and $\pi_2(K)$ are both compact
2. If $\pi_1(K)$ and $\pi_2(K)$ are both compact, then K is compact
3. If K is connected, then both $\pi_1(K)$ and $\pi_2(K)$ are connected
4. If $\pi_1(K)$ and $\pi_2(K)$ are both connected, then K is connected

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **3398932147**
Option 1 ID : **3398938357**
Option 2 ID : **3398938358**
Option 3 ID : **3398938359**
Option 4 ID : **3398938360**
Status : **Not Answered**
Chosen Option : --

Q.31 Consider the problem of minimizing /maximizing

$$J[y] = \int_0^{\pi/8} (y'^2 + 2yy' - 16y^2)dx,$$

subject to $y(0) = 0,\ y\left(\frac{\pi}{8}\right) = 1.$ Then

1. every extremizer of J is a minimizer

2. every extremizer of J is a maximizer

3. there exists an extremizer of J which is not a minimizer

4. there exists an extremizer of J which is not a maximizer

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932156
Option 1 ID : 3398938393
Option 2 ID : 3398938394
Option 3 ID : 3398938395
Option 4 ID : 3398938396
Status : Not Answered
Chosen Option : --

Q.32 Consider the integration formula

$\int_{x_0}^{x_1} f(x)dx = \frac{h}{2}[f(x_0) + f(x_1)] + ph^2(f'(x_0) - f'(x_1)),$

where $h = x_1 - x_0$. Then the constant p such that the above formula gives the exact value for the highest degree polynomial and the degree d of the corresponding polynomial are given by

1. $p = \frac{1}{6}, d = 4$
2. $p = \frac{1}{12}, d = 3$
3. $p = \frac{1}{6}, d = 3$
4. $p = \frac{1}{12}, d = 4$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932153
Option 1 ID : 3398938381
Option 2 ID : 3398938382
Option 3 ID : 3398938383
Option 4 ID : 3398938384
Status : Not Answered
Chosen Option : --

Q.33 If $u(x, y)$ is the solution of the following problem

$u\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 1$, such that $u(x_0(s), y_0(s)) = u_0(s)$, where

$x_0(s) = s,\ y_0(s) = s,\ u_0(s) = {}^{s}/_{2}$. Then $u(2,1)$ is equal to

1. 1/2
2. −(1/2)
3. 1
4. −1

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932151
Option 1 ID : 3398938373
Option 2 ID : 3398938374
Option 3 ID : 3398938375
Option 4 ID : 3398938376
Status : Not Answered
Chosen Option : --

Q.34 Let $B(0,1) = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$. Consider $u \in C^2(\overline{B(0,1)})$, satisfying $\Delta u + \lambda u = 0$ in $B(0,1)$, $\frac{\partial u}{\partial n} = 0$ on $\partial(B(0,1))$. Then

1. $\lambda \geq 0$
2. $\lambda < 0$
3. $\lambda = 0$ implies $u = 0$
4. $\lambda = 0$ implies u is a constant

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932152
Option 1 ID : 3398938377
Option 2 ID : 3398938378
Option 3 ID : 3398938379
Option 4 ID : 3398938380
Status : Not Answered
Chosen Option : --

Q.35 Let $f: \mathbb{R}^2 \to (-\infty, 0]$ be such that $f(0,0) = 0$. Consider the following system of ODEs

$x'(t) = f(x(t), y(t)), \quad y'(t) = y^2(t), t > 0.$

A set $S \subseteq \mathbb{R}^2$ is said to be positively invariant set if the following holds. Assume that $(x(0), y(0)) \in S$. Then for $t \geq 0$, if the solution $(x(t), y(t))$ exists, then $(x(t), y(t)) \in S$. Then which of the following sets are positively invariant

1. $\{(x,y) \in \mathbb{R}^2 : x \leq 0, y \leq 0\}$
2. $\{(x,y) \in \mathbb{R}^2 : y \geq 0\}$
3. $\{(x,y) \in \mathbb{R}^2 : x \leq 0, y \leq -1\}$
4. $\{(x,y) \in \mathbb{R}^2 : x \leq 0, y \geq 0\}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932149
Option 1 ID : 3398938365
Option 2 ID : 3398938366
Option 3 ID : 3398938367
Option 4 ID : 3398938368
Status : Not Answered
Chosen Option : --

Q.36 Is Jacobi's necessary condition satisfied by the admissible extremal for

$$J[y] = \int_0^b (y'^2 - y^2)\,dx\,?$$

1. Yes, for all values of b
2. No, for any value of b
3. Yes, for $b < \pi$
4. Yes, for $b \geq \pi$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932155
Option 1 ID : 3398938389
Option 2 ID : 3398938390
Option 3 ID : 3398938391
Option 4 ID : 3398938392
Status : Not Answered
Chosen Option : --

Q.37 Consider a circular disc in the xy-plane with the center at (0,0) and radius one. A simple pendulum of length ℓ and mass m is attached to the rim of the disc. Suppose the disc is rotating with a constant angular velocity ω about the origin and the pendulum is moving in the same plane. Assume that the gravitational force is acting along the positive y-axis and $\theta(t)$ denotes the angle of deviation of the pendulum from the mean position at time t. Then a Lagrangian of the system is

1. $L=\frac{1}{2}ml^2\dot{\theta}^2+ml\omega^2\sin(\dot{\theta}t-\omega t)+mgl\cos\theta$

2. $L=\frac{1}{2}ml^2\dot{\theta}^2+ml\omega\dot{\theta}\sin(\theta-\omega t)+mgl\cos\theta$

3. $L=\frac{1}{2}m(l^2\dot{\theta}^2+\omega^2)+ml\omega\dot{\theta}\sin(\theta-\omega t)+mgl\cos\theta-mg\sin\omega t$

4. $L=\frac{1}{2}ml^2\dot{\theta}^2+ml\omega\dot{\theta}^2\sin(\dot{\theta}t-\omega t)+mgl\sin\theta$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932159
Option 1 ID : 3398938405
Option 2 ID : 3398938406
Option 3 ID : 3398938407
Option 4 ID : 3398938408
Status : Not Answered
Chosen Option : --

Q.38 Let $K:[a,b]\times[a,b]\to\mathbb{R}$ be a nonzero continuous map with $K(x,t)=K(t,x)$, for $t,x\in[a,b]$ and $I:C[a,b]\to C[a,b]$ be the identity operator. Define $T,S:C[a,b]\to C[a,b]$ such that for $\varphi\in C[a,b], x\in[a,b]$

$$(T\varphi)(x)=\int_a^x K(x,t)\varphi(t)dt,(S\varphi)(x)=\int_a^b K(x,t)\varphi(t)dt.$$

Then which of the following statements are true?

1. $\forall\lambda\in\mathbb{R}\setminus\{0\}$, the only solution of $(I-\lambda T)\varphi=0$ is $\varphi\equiv 0$

2. $\forall\lambda\in\mathbb{R}\setminus\{0\},\forall f\in C[a,b]$, there exists a unique solution of $(I-\lambda T)\varphi=f$

3. $\forall\lambda\in\mathbb{R}\setminus\{0\},\forall f\in C[a,b]$, there exists a unique solution of $(I-\lambda S)\varphi=f$

4. $\exists\lambda\in\mathbb{R}\setminus\{0\}$ such that $(I-\lambda S)\varphi=0$ has a nonzero solution

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932157
Option 1 ID : 3398938397
Option 2 ID : 3398938398
Option 3 ID : 3398938399
Option 4 ID : 3398938400
Status : Not Answered
Chosen Option : --

Q.39 Consider the first order initial value problem $y'(x)=-y(x), x>0, y(0)=1$ and the corresponding numerical scheme $4\left(\frac{y_{n+1}-y_{n-1}}{2h}\right)-3\left(\frac{y_{n+1}-y_n}{h}\right)=-y_n$, with $y_0=1, y_1=e^{-h}$, where h is the step size. Then which of the following statements are true?

1. The order of the scheme is 1

2. The order of the scheme is 2

3. $|y_n-y(nh)|\to\infty$ as $n\to\infty$

4. $|y_n-y(nh)|\to 0$ as $n\to\infty$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932154
Option 1 ID : 3398938385
Option 2 ID : 3398938386
Option 3 ID : 3398938387
Option 4 ID : 3398938388
Status : Not Answered
Chosen Option : --

Q.40 Consider the integral equation

$$\varphi(x) - \lambda \int_0^{2\pi} \cos(x-t)\varphi(t)dt = f(x), x \in [0,2\pi],$$

where $f \in C[0,2\pi]$.

Then which of the following statements are true?

1. If $\lambda = 10$ and $f(x) = \cos x$, then the integral equation has a solution
2. If $\lambda = \frac{1}{\pi}$ and $f(x) = \cos x$, then the integral equation has no solution
3. For every $\lambda \in \mathbb{R}$ and $f(x) = \cos 2x$, the integral equation has a solution
4. For every $\lambda \in \mathbb{R}$ and $f(x) = \sin 2x$, the integral equation has unique solution

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932158
Option 1 ID : 3398938401
Option 2 ID : 3398938402
Option 3 ID : 3398938403
Option 4 ID : 3398938404
Status : Answered
Chosen Option : 1,2

Q.41 Let $p \in C^1[0,1], q \in C[0,1]$ and p be positive. Consider a solution $x > 0$ of the boundary value problem

$p(t)x''(t) + p'(t)x'(t) + q(t)x(t) = \beta x(t), t \in (0,1), x'(0) = 0 = x'(1)$, where $\beta \in \mathbb{R}$. If $\int_0^1 q(t)dt > 0$ then which of the following statements are true?

1. $\beta \in (-4,-1]$
2. $\beta \in (-1,0]$
3. $\beta \in (0,\infty)$
4. $\beta \in (-\infty,-4]$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932150
Option 1 ID : 3398938369
Option 2 ID : 3398938370
Option 3 ID : 3398938371
Option 4 ID : 3398938372
Status : Not Answered
Chosen Option : --

Q.42 Let $g: \mathbb{R} \to \mathbb{R}$ be locally Lipschitz and $xg(x) \le 0, \forall x \in \mathbb{R}$. Then which of the following statements are true for the initial value problem

$x'(t) = g(x(t)), \quad x(0) = x_0 \in \mathbb{R}.$

1. There exists a local solution
2. For any $x_0 \in \mathbb{R}$, the solution exists on $[0, \infty)$
3. There is a finite time blow-up for some x_0
4. All the solutions are bounded on $[0, \infty)$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932148
Option 1 ID : 3398938361
Option 2 ID : 3398938362
Option 3 ID : 3398938363
Option 4 ID : 3398938364
Status : Not Answered
Chosen Option : --

Q.43 Consider a LP problem:

Maximize $z = \underline{c}'\underline{x}$ subject to $A\underline{x} \le \underline{b}$, $\underline{x} \ge \underline{0}$.

Which of the following are true?

1. If the primal has an optimal solution, then the dual also has an optimal solution.
2. The primal can have an optimal solution and the dual may have no feasible solution.
3. The dual can have an optimal solution and the primal may have no feasible solution.
4. If the primal has no feasible solution then the dual also has no feasible solution.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932176
Option 1 ID : 3398938473
Option 2 ID : 3398938474
Option 3 ID : 3398938475
Option 4 ID : 3398938476
Status : Not Answered
Chosen Option : --

Q.44 For a 2^4 factorial experiment with 4 treatments A, B, C, D each at 2 levels, 4 blocks of size 4 each were available. If the key block is

(1) (bc) (acd) (abd)

which of the following treatment combinations are confounded?

1. ABC
2. ABD
3. ACD
4. BCD

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932174
Option 1 ID : 3398938465
Option 2 ID : 3398938466
Option 3 ID : 3398938467
Option 4 ID : 3398938468
Status : Not Answered
Chosen Option : --

Q.45 Consider two samples of sizes n_1 and n_2 drawn from a finite population of size $N \geq 10$, using SRSWR and SRSWOR respectively. Suppose population coefficients of variation of the two sample means are the same. Assume that the population mean is positive. Which of the following are true?

1. $n_1 \geq n_2$
2. If $n_1 = n_2$, then $n_2 = 1$
3. $N(n_1 - n_2) = n_2(n_1 - 1)$
4. $n_2(N-1) = n_1(N - n_2)$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932173
Option 1 ID : 3398938461
Option 2 ID : 3398938462
Option 3 ID : 3398938463
Option 4 ID : 3398938464
Status : Not Answered
Chosen Option : --

Q.46 Consider a 2-way analysis of variance experiment with 4 rows, 3 columns and a single observation per cell. The experiment was carried out keeping cells (1,1), (2,2), (3,2) and (4,1) empty. Let α_i, $i = 1,2,3,4$ be the row effects and β_j, $j = 1,2,3$ be the column effects. Which of the following statements are true?

1. Only 2 linearly independent elementary contrasts of α are estimable
2. the elementary contrast $\beta_1 - \beta_2$ is estimable
3. the contrast $\alpha_1 - \alpha_2 + \alpha_3 - \alpha_4$ is not estimable
4. the contrast $\alpha_1 - \alpha_3$ is estimable

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932171
Option 1 ID : 3398938453
Option 2 ID : 3398938454
Option 3 ID : 3398938455
Option 4 ID : 3398938456
Status : Not Answered
Chosen Option : --

Q.47 X is a random variable with normal distribution having mean 1 and variance 4. Which of the following are true?

1. $E[X^2] = 18$
2. $\text{Var}(X^2) = 48$
3. $E[\min(X,3)] < 1$
4. $E[(X+1)^2] \leq E[(X-1)^2]$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932164
Option 1 ID : 3398938425
Option 2 ID : 3398938426
Option 3 ID : 3398938427
Option 4 ID : 3398938428
Status : Not Answered
Chosen Option : --

Q.48 Let $X_1, X_2, \ldots, X_m$ be a random sample from a continuous distribution F and let $Y_1, \ldots, Y_n$ be a random sample from a continuous distribution G. We want to test the hypothesis $H_0: F(x) = G(x)$ $\forall x$ versus $H_1: G(x) = F(x-\theta), \forall x$, for some fixed θ ($\neq 0$). Assume that $X_1, \ldots, X_m, Y_1, \ldots, Y_n$ are independent. Let R_{m+j} denote the rank of Y_j in the combined ordering and $S = \sum_{j=1}^{n} R_{m+j}$. Which of the following are true?

1. Under H_0, $P[R_{m+j} = k] = \frac{1}{n+m}$ for $k = 1,2,\cdots,n+m$
2. If the observed value of S is $\frac{n(n+m+1)}{2}$, then the large sample test does

3. If $\theta > 0$, a large value of S supports H_1

4. The assumption $n = m$ is needed to obtain the asymptotic null distribution of $\frac{S - E_{H_0}[S]}{\sqrt{\text{Var}_{H_0}(S)}}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932169
Option 1 ID : 3398938445
Option 2 ID : 3398938446
Option 3 ID : 3398938447
Option 4 ID : 3398938448
Status : Not Answered
Chosen Option : --

Q.49 Suppose the hazard rate function of the lifetime T of a system is $h(t) = t^2$ for $t > 0$. Then which of the following are true?

1. The probability that the system fails prior to 3 time units is $\frac{1}{9}$

2. The probability that the system fails at time $\frac{1}{3}$ given $T \geq \frac{1}{3}$ is $\frac{1}{9}$

3. The probability density function of T is $f(x) = t^2 e^{-(t^3/3)}$ for $t > 0$

4. $E[T^3] = 3$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932175
Option 1 ID : 3398938469
Option 2 ID : 3398938470
Option 3 ID : 3398938471
Option 4 ID : 3398938472
Status : Not Answered
Chosen Option : --

Q.50 Let N_t be the number of customers that enter my shop up-to time t. Assume that $(N_t)_{t\geq 0}$ is Poisson process with parameter $\lambda = 5$. Whenever a customer enters, a fair die (usual die with six faces) is rolled. Let X be the number of 1's up-to time 10 and Y be the number of 2's up-to time 10. Which of the following are correct?

1. X, Y are independent

2. $X + Y \leq 50$

3. The joint distribution of (X, Y) is multinomial

4. X, Y are Poisson random variables

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932177
Option 1 ID : 3398938477
Option 2 ID : 3398938478
Option 3 ID : 3398938479
Option 4 ID : 3398938480
Status : Not Answered
Chosen Option : --

Q.51 Which of the following conditions imply that $X_n \to X$ in distribution as $n \to \infty$?

1. $X_n^2 \to X^2$ with probability one
2. $E[|X_n - X|^2] \to 0$
3. $E[e^{i\lambda X_n}] \to E[e^{i\lambda X}]$ for all $\lambda \in \mathbb{R}$
4. $X_n \to X$ with probability one

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932160
Option 1 ID : 3398938409
Option 2 ID : 3398938410
Option 3 ID : 3398938411
Option 4 ID : 3398938412
Status : Not Answered
Chosen Option : --

Q.52 Let X and Y be two random variables on a probability space. Consider the following statements

A. Each of X and Y is normally distributed

B. X and Y are independent

C. The conditional distribution of X given $Y = y$ and the conditional distribution of Y given $X = x$ are normal for all x and y.

Then for (X, Y) to have a bivariate normal distribution

1. A alone is sufficient
2. A and B together are sufficient
3. A and B are necessary
4. C is necessary

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932172
Option 1 ID : 3398938457
Option 2 ID : 3398938458
Option 3 ID : 3398938459
Option 4 ID : 3398938460
Status : Not Answered
Chosen Option : --

Q.53 P is the transition matrix of a finite state space Markov chain. Which of the following statements are necessarily true? (Below I denotes the identity matrix of the same size as P).

1. If P is irreducible then P^2 is irreducible
2. If P is not irreducible then P^2 is not irreducible
3. If P is irreducible then $\frac{I+P}{2}$ is irreducible
4. If P is irreducible and aperiodic, then P^3 is irreducible

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932162
Option 1 ID : 3398938417
Option 2 ID : 3398938418
Option 3 ID : 3398938419
Option 4 ID : 3398938420
Status : Not Answered
Chosen Option : --

Q.54 Let $X_1, X_2, \ldots, X_n$ be i.i.d. from Poisson (λ) where $\lambda > 0$. Consider testing $H_0: \lambda \le \lambda_0$ versus $H_1: \lambda > \lambda_0$.

Let Test A be the test that rejects H_0 if $\sum_{i=1}^{n} X_i > k$ where k is such that $P(\sum_{i=1}^{n} X_i > k \mid \lambda = \lambda_0) = \alpha$.

Let Test B be the test that rejects H_0 if $\sum_{i=1}^{n} X_i < k$ where k is such that $P(\sum_{i=1}^{n} X_i < k \mid \lambda = \lambda_0) = \alpha$.
Then,

1. Test A is the likelihood ratio test of size α
2. Test B is the likelihood ratio test of size α
3. Test A is a UMP test of size α
4. Test B is a UMP test of size α

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932167
Option 1 ID : 3398938437
Option 2 ID : 3398938438
Option 3 ID : 3398938439
Option 4 ID : 3398938440
Status : Not Answered
Chosen Option : --

Q.55 Let $X_1, X_2, \ldots, X_n$ be a random sample from an exponential (λ) distribution, $\lambda > 0$, i.e.,

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } \; x > 0 \\ 0 & \text{otherwise} \end{cases}$$

Let $\overline{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$. Then which of the following are correct?

1. $\overline{X}_n$ is MLE of λ
2. $\frac{1}{\overline{X}_n}$ is MLE of λ
3. $\frac{1}{n}\left(\frac{1}{X_1} + \frac{1}{X_2} + \cdots + \frac{1}{X_n}\right)$ is MLE of λ
4. $\frac{1}{\overline{X}_n}$ is an unbiased estimator of λ

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932166
Option 1 ID : 3398938433
Option 2 ID : 3398938434
Option 3 ID : 3398938435
Option 4 ID : 3398938436
Status : Not Answered
Chosen Option : --

Q.56 Suppose Y_1 and Y_2 are i.i.d. Uniform$(0, \theta)$, where $\theta > 0$. Define $M = \text{Max}\{Y_1, Y_2\}$. Which of the following are confidence intervals for θ with confidence coefficient 0.91?

1. $\left[M, \frac{10}{3}M\right]$
2. $\left[\frac{9}{10}M, \frac{10}{3}M\right]$
3. $\left[\frac{1}{2}M, \frac{10}{3}M\right]$
4. $\left[\frac{1}{2}M, 2M\right]$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932168
Option 1 ID : 3398938441
Option 2 ID : 3398938442
Option 3 ID : 3398938443
Option 4 ID : 3398938444
Status : Not Answered
Chosen Option : --

Q.57 Let $X_1, X_2, \ldots$ be i.i.d. random variables from a distribution F. Let
$Y_n = \min\{X_1, \cdots, X_n\}$.
If nY_n converges in distribution to exponential distribution with mean 1/2, then F could be

1. $N(0,1)$
2. $\text{Uniform}(0,1)$
3. $\text{Uniform}(0,\frac{1}{2})$
4. $\text{Uniform}(0,2)$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932163
Option 1 ID : 3398938421
Option 2 ID : 3398938422
Option 3 ID : 3398938423
Option 4 ID : 3398938424
Status : Not Answered
Chosen Option : --

Q.58 Let (Y_i, X_i) $i = 1, \ldots, n$ be observations where X_i are the regressers and Y_i are response variables which follow Bernoulli distribution with probability of success $\pi(X_i)$, $i = 1, \ldots, n$. Suppose the log-odds ratio is a linear function with intercept α and slope β. Which of the following statements are true?

1. If $\beta > 0$ then $\pi(x)$ is an increasing function of x
2. $E[Y \mid X] = e^{\alpha+\beta x}$
3. If $x = -\frac{\alpha}{\beta}$ with $\beta \neq 0$, then $\pi(x) = \frac{1}{2}$
4. If $x = -\frac{\alpha}{\beta}$ then $\pi(x+c) = \pi(x-c)$ for any c.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932170
Option 1 ID : 3398938449
Option 2 ID : 3398938450
Option 3 ID : 3398938451
Option 4 ID : 3398938452
Status : Not Answered
Chosen Option : --

Chosen Option : --

Q.59 Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables with probability mass function given by $p_\theta(x) = \left(\frac{\theta}{2}\right)^{|x|} (1-\theta)^{1-|x|}$ for $x = -1, 0, 1$, where $\theta \in (0,1)$. Then

1. $\sum_{i=1}^{n} |X_i|$ is complete sufficient for θ
2. $\frac{1}{n}\sum_{i=1}^{n} X_i$ is MLE of θ
3. $\frac{1}{n}\sum_{i=1}^{n} |X_i|$ is MLE of θ
4. $\frac{1}{n}\sum_{i=1}^{n} X_i$ is unbiased for θ

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932165
Option 1 ID : 3398938429
Option 2 ID : 3398938430
Option 3 ID : 3398938431
Option 4 ID : 3398938432
Status : Not Answered
Chosen Option : --

Q.60 Let $X_1, X_2, \ldots$ be i.i.d. with mean μ and variance $\sigma^2 > 0$. Let $Z_n = \frac{(X_1 + \ldots + X_n)^2}{n}$. Then which of the following are true?

1. $Z_n \to \mu^2$ with probability one
2. If $\mu \neq 0$ then $Z_n \to \infty$ with probability one
3. If $\mu = 0$ then $\frac{Z_n}{\sigma^2}$ converges in distribution
4. If $\mu = 0$ then $\frac{Z_n}{\sigma^2}$ converges in distribution to standard normal

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ
Question ID : 3398932161
Option 1 ID : 3398938413
Option 2 ID : 3398938414
Option 3 ID : 3398938415
Option 4 ID : 3398938416
Status : Not Answered
Chosen Option : --